What’s wrong with RANS?

or

How to read results of RANS (CFD) simulations?

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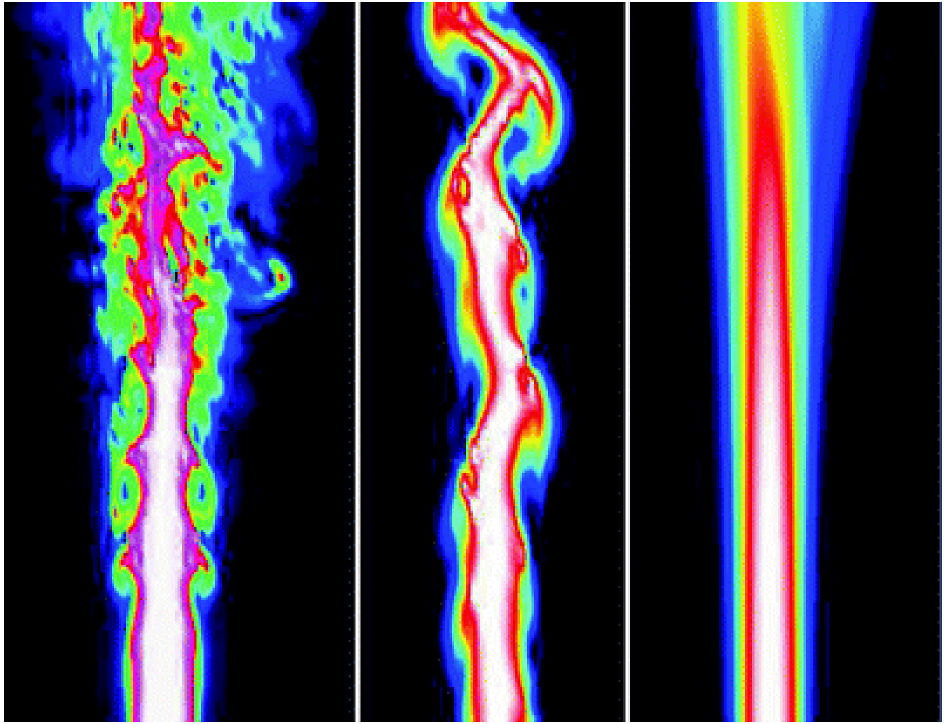
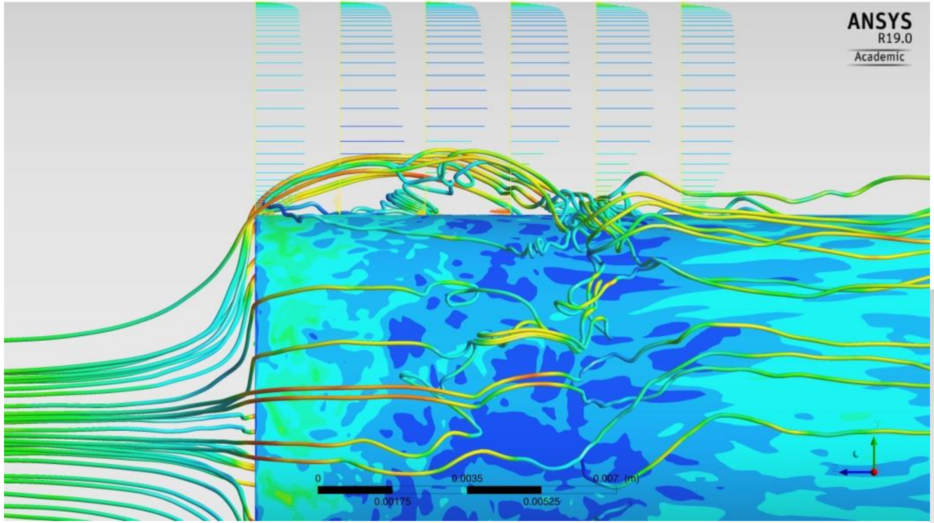
DNS/LES/RANS - the difference



- DNS (left), LES (middle) and RANS (right) predictions of a turbulent jet. LES requires less computational effort than DNS, while delivering more detail than the inexpensive RANS

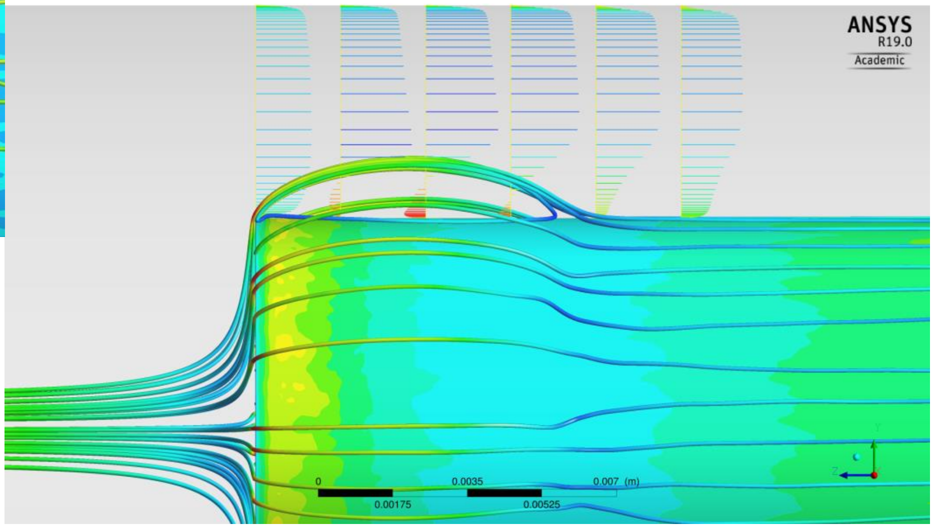
Source: Maries A. et al. (2012), DOI: 10.1007/978-3-642-27343-8_7

LES and RANS

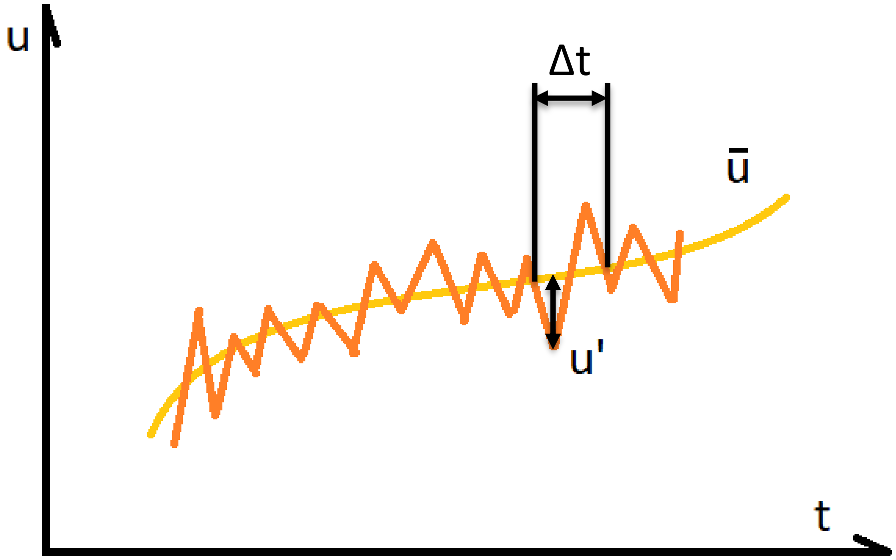


- LES (selected instant)

- LES (time-averaged) ≅ RANS



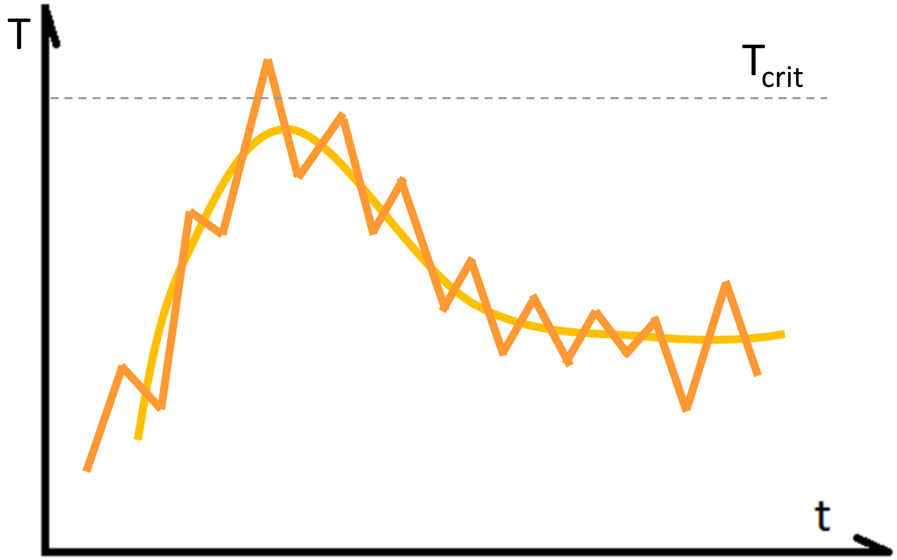
Time averaging – the essence of RANS



$$u = \bar{u} + u'$$

$$\bar{u} = \frac{1}{\Delta t}\int_{t}^{t+\Delta t} u dt$$

Time averaging – the essence of RANS

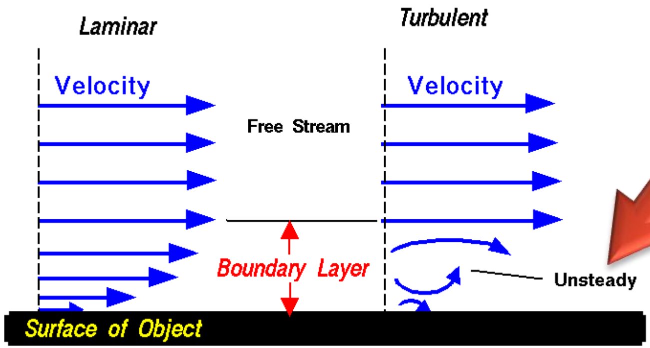


- Averaging may lead to non-conservative conclusions!

Navier Stokes equations

$$
\begin{cases}
\dfrac{\partial \rho}{\partial t} = \dfrac{\partial(\rho u_x)}{\partial x} + \dfrac{\partial(\rho u_y)}{\partial y} + \dfrac{\partial(\rho u_z)}{\partial z} \\[2ex]
\dfrac{\partial(\rho u_x)}{\partial t} + \dfrac{\partial(\rho u_x{}^2)}{\partial x} + \dfrac{\partial(\rho u_x u_y)}{\partial y} + \dfrac{\partial(\rho u_x u_z)}{\partial z} = -\dfrac{\partial p}{\partial x} + \dfrac{1}{Re}\left(\dfrac{\partial \tau_{xx}}{\partial x} + \dfrac{\partial \tau_{xy}}{\partial y} + \dfrac{\partial \tau_{xz}}{\partial z}\right) \\[2ex]
\dfrac{\partial(\rho u_y)}{\partial t} + \dfrac{\partial(\rho u_x u_y)}{\partial x} + \dfrac{\partial(\rho u_y{}^2)}{\partial y} + \dfrac{\partial(\rho u_y u_z)}{\partial z} = -\dfrac{\partial p}{\partial y} + \dfrac{1}{Re}\left(\dfrac{\partial \tau_{xy}}{\partial x} + \dfrac{\partial \tau_{yy}}{\partial y} + \dfrac{\partial \tau_{yz}}{\partial z}\right) \\[2ex]
\dfrac{\partial(\rho u_z)}{\partial t} + \dfrac{\partial(\rho u_x u_z)}{\partial x} + \dfrac{\partial(\rho u_y u_z)}{\partial y} + \dfrac{\partial(\rho u_z{}^2)}{\partial z} = -\dfrac{\partial p}{\partial z} + \dfrac{1}{Re}\left(\dfrac{\partial \tau_{xz}}{\partial x} + \dfrac{\partial \tau_{yz}}{\partial y} + \dfrac{\partial \tau_{zz}}{\partial z}\right)
\end{cases}
$$

Reynolds Shear Stress Tensor



Velocity is zero at the surface (no – slip)

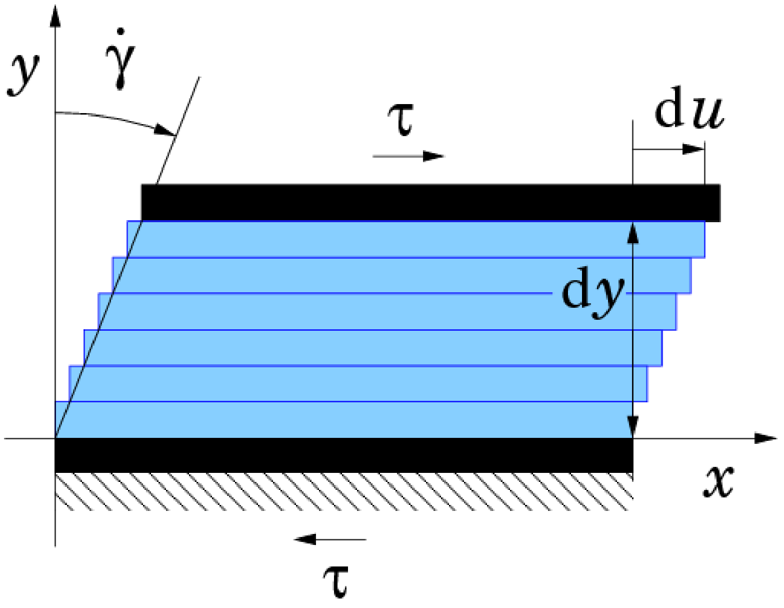
Source: www.grc.nasa.gov/www/k-12/airplane/boundlay.html

$$Re = \frac{\rho U D_h}{\mu}$$

$$\tau_{ij} = -\rho \overline{u_i{}' u_j{}'}$$

They are all made of layers

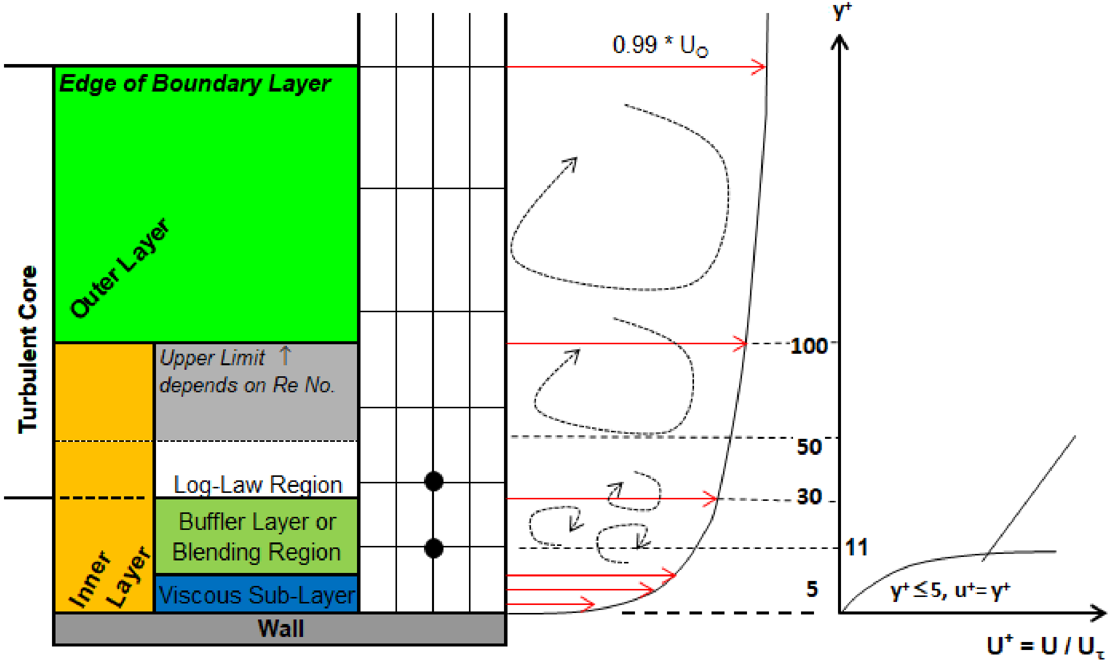
Source: knowyourmeme.com/photos/1490591-shrek



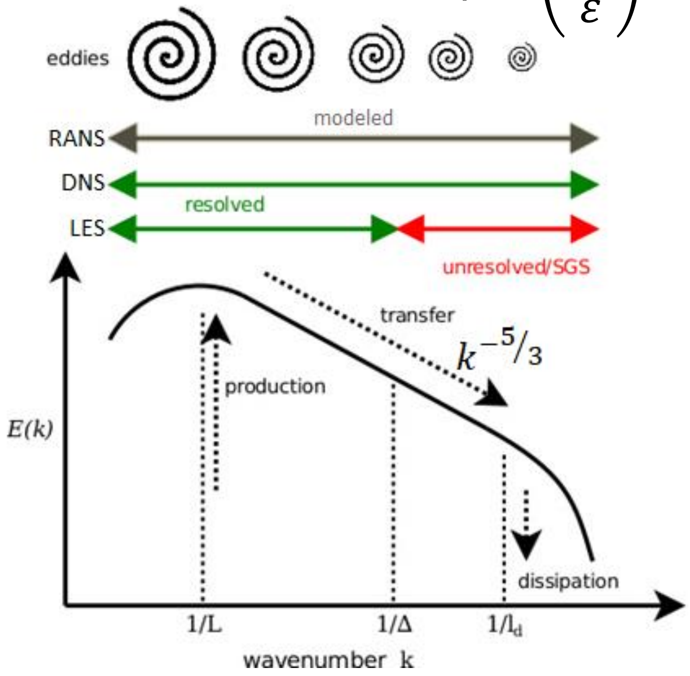
Source: de.wikipedia.org/wiki/Schergeschwindigkeit

Boundary Layer and Energy Cascade

$$\eta = \left(\frac{\nu^3}{\varepsilon}\right)^{1/4}$$



Source: www.cfdyna.com/CFDHT/turbulenceCFD.html

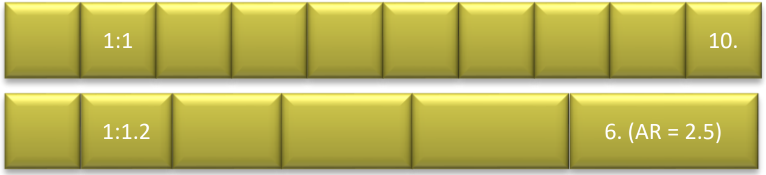


Based on source: www.pgrete.de/research

Mesh requirements

Approach	AR	Physical shape
RANS*	75	
LES*	15	
DNS	1	

(*) 1.1 < Cell Layer Growth Rate < 1.3



2D: 100 3D: 1000

2D: 36 3D: 216

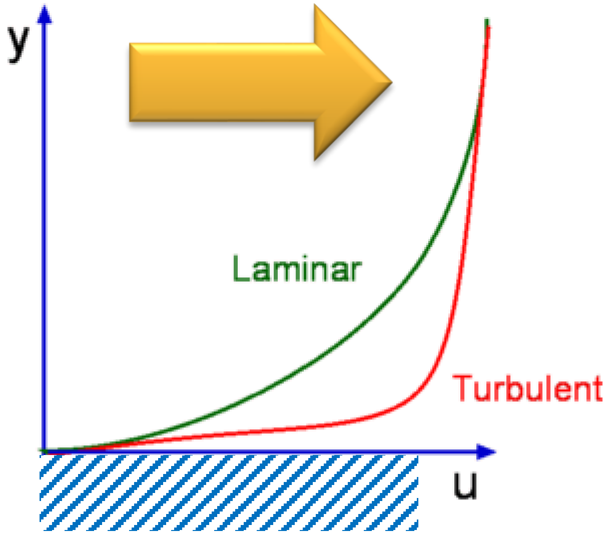
Boussinesq closure model

- So instead of tensor, like this:

$$\left(\frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z}\right)$$
$$\left(\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} + \frac{\partial \tau_{yz}}{\partial z}\right)$$
$$\left(\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \tau_{zz}}{\partial z}\right)$$

- In RANS, one solves this*:

$$\tau_{ij} = \tau^{lam} + \tau_{ij}{}^{turb}$$



Based on source: www-mdp.eng.cam.ac.uk/web/library/enginfo/aerothermal_dvd_only/aero/fprops/introvisc/node8.html

Boussinesq closure model

- So instead of tensor, like this:

$$\begin{matrix} \left(\frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z}\right) \\ \left(\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} + \frac{\partial \tau_{yz}}{\partial z}\right) \\ \left(\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \tau_{zz}}{\partial z}\right) \end{matrix}$$

- In RANS, one solves this*:

$$\tau_{ij} = \tau^{lam} + \tau_{ij}{}^{turb} \qquad \tau^{lam} = \mu \frac{d\bar{u}}{dx_y}$$

$$\tau_{ij}{}^{turb} \equiv -\rho \overline{u_i{}' u_j{}'} = \mu_t \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) - \frac{2}{3}\left(\rho k + \mu_t \frac{\partial u_k}{\partial x_k}\right)\delta_{ij}$$

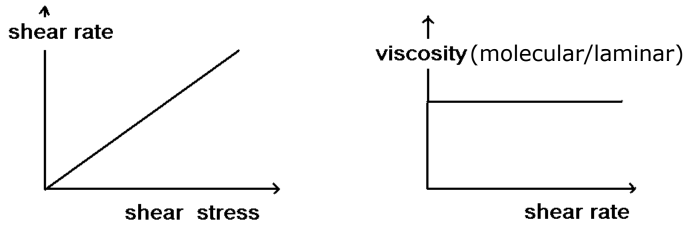
shear stress   shear rate

„(…) The Eddie Viscosity hypothesis was posed for making the things simpler, in the sense that the turbulent Reynolds stresses (which are ugly and nonlinear in velocity perturbations) are simplified to be proportional to the gradients of the mean velocity, as happens in Newtonian laminar flows with the viscous stresses. The coefficient of proportionality is termed the Eddie Viscosity, which far from being a constant or fluid property, is a magnitude dependant on the flow field and its solution.

The Eddie viscosity hypothesis is inherently wrong, in that the Reynolds stresses are in general not co-linear with the mean velocity gradients, as has being discovered by DNS solutions. However, the numerical methods stemming from this simplification (such as RANS methods) are low-time consuming and can be used, sometimes massively, by computational fluid dynamists to obtain approximate solutions of turbulent flows.” (Clausius2)

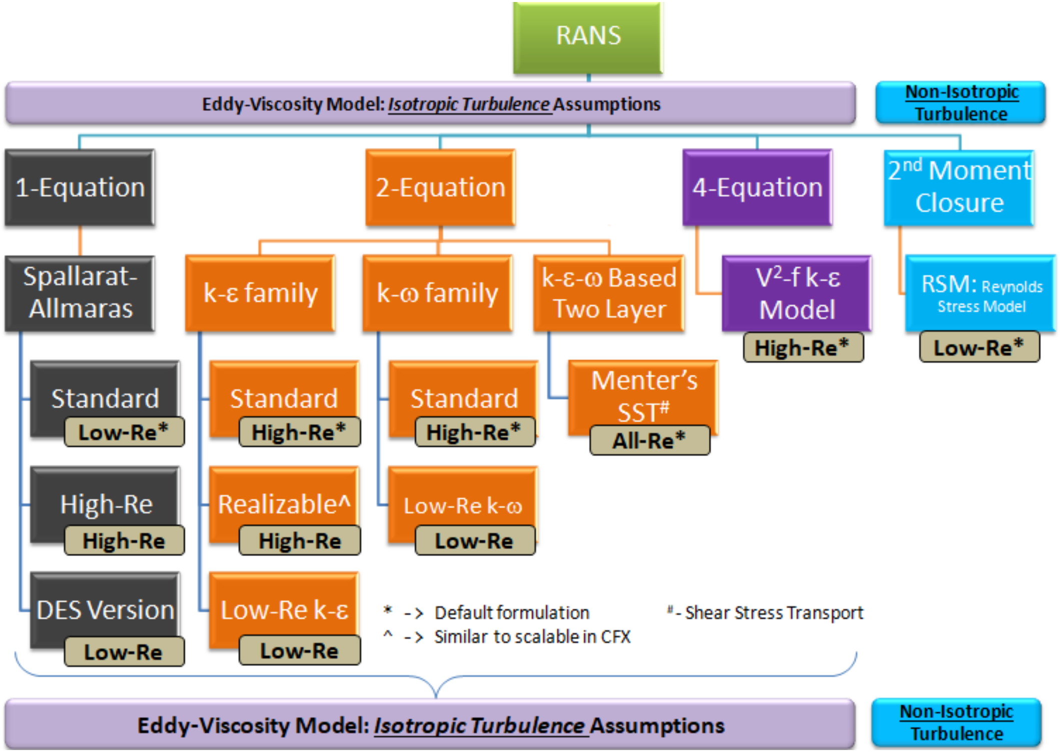
Source: https://www.physicsforums.com/threads/moleculer-viscosity-eddy-viscosity.190265/

Newtonian fluid @ constant temperature



Source: people.sju.edu/~phabdas/physics/rheo.html

RANS models family



Source: www.cfdyna.com/CFDHT/turbulenceCFD.html

RANS example - realizable k-ε model (so-called Shih model '94)

Invented by NASA

Implemented by ANSYS

$$\mu_t = \rho C_\mu \frac{k^2}{\varepsilon}$$

$$\frac{\partial}{\partial t}(\rho k) + \frac{\partial}{\partial x_j}(\rho k u_j) = \frac{\partial}{\partial x_j}\left[\left(\mu + \frac{\mu_t}{\sigma_k}\right)\frac{\partial k}{\partial x_j}\right] + G_k + G_b - \rho\varepsilon - Y_M + S_k$$

$$\frac{\partial}{\partial t}(\rho \varepsilon) + \frac{\partial}{\partial x_j}(\rho \varepsilon u_j) = \frac{\partial}{\partial x_j}\left[\left(\mu + \frac{\mu_t}{\sigma_\varepsilon}\right)\frac{\partial \varepsilon}{\partial x_j}\right] + \rho C_1 S\varepsilon - \rho C_2 \frac{\varepsilon^2}{k + \sqrt{\nu\varepsilon}} + C_{1\varepsilon}\frac{\varepsilon}{k} C_{3\varepsilon} G_b + S_\varepsilon$$

G_k generation of turbulence kinetic energy due to the mean velocity gradients,
G_b generation of turbulence kinetic energy due to buoyancy,
Y_M represents the contribution of the fluctuating dilatation in compressible turbulence to the overall dissipation rate,
S modulus of the mean rate-of-strain tensor

$$C_1 = \max\left[0.43, \frac{Sk}{Sk + 5\varepsilon}\right]$$

C_2 and $C_{1\varepsilon}$ are constants,

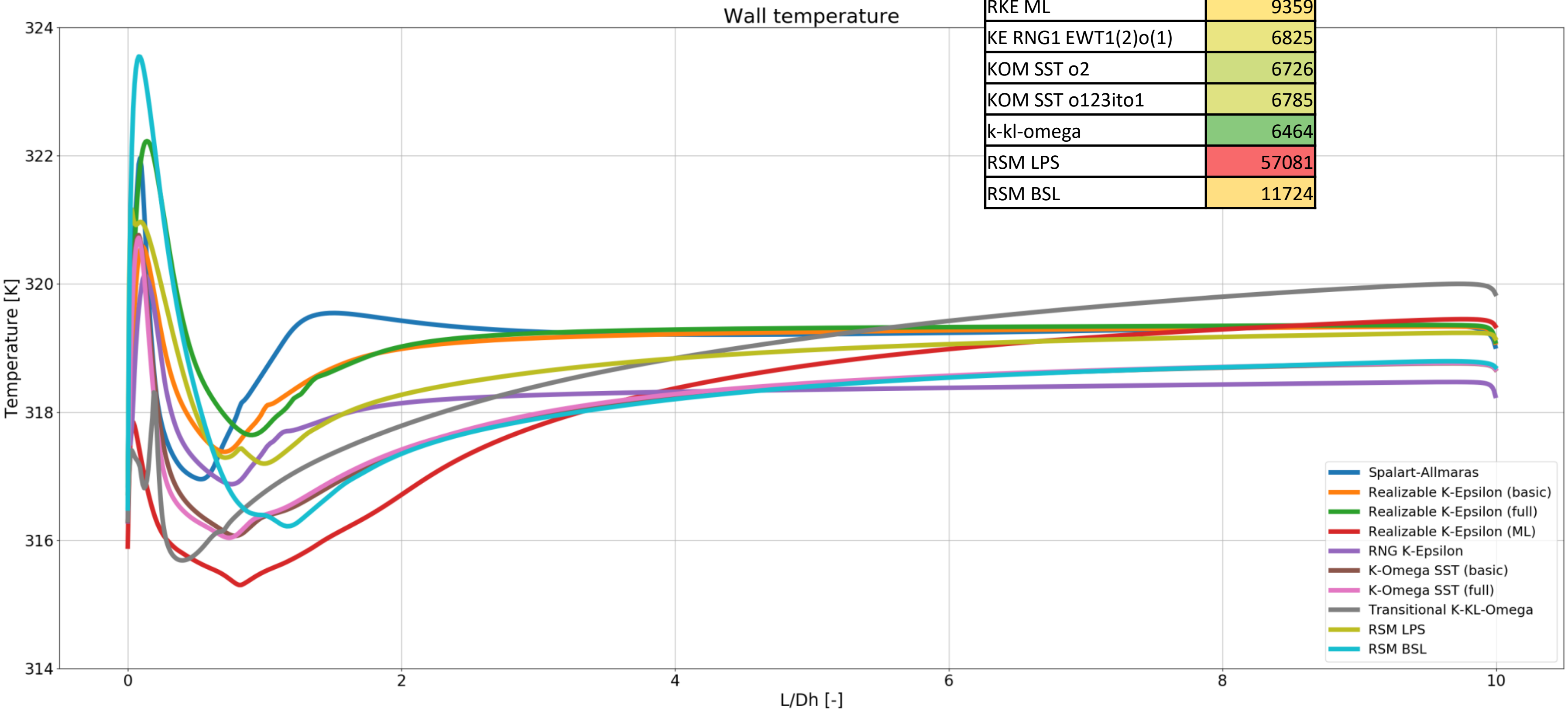
$$C_{3\varepsilon} = \tanh\left|\frac{u_{\parallel g}}{u_{\perp g}}\right|$$

σ_k and σ_ε are the turbulent Prandtl numbers,
S_k and S_ε are user-defined source terms.

$$k = \frac{1}{2}\left(\overline{(u_x{}')^2} + \overline{(u_y{}')^2} + \overline{(u_z{}')^2}\right)$$

Results spread

RANS model	iterations
SA VB VH	6322
RKE EWT	6976
RKE EWT1o23	8541
RKE ML	9359
KE RNG1 EWT1(2)o(1)	6825
KOM SST o2	6726
KOM SST o123ito1	6785
k-kl-omega	6464
RSM LPS	57081
RSM BSL	11724



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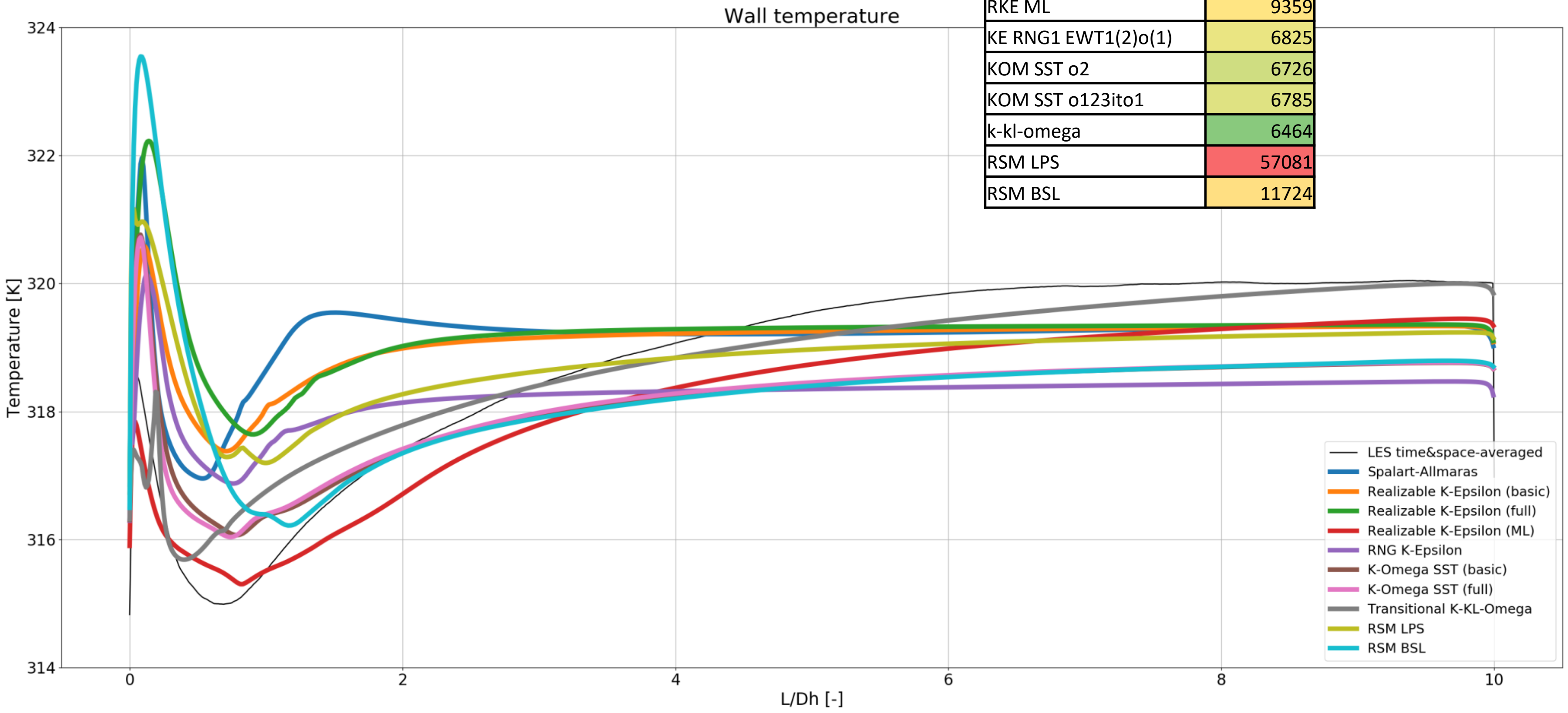

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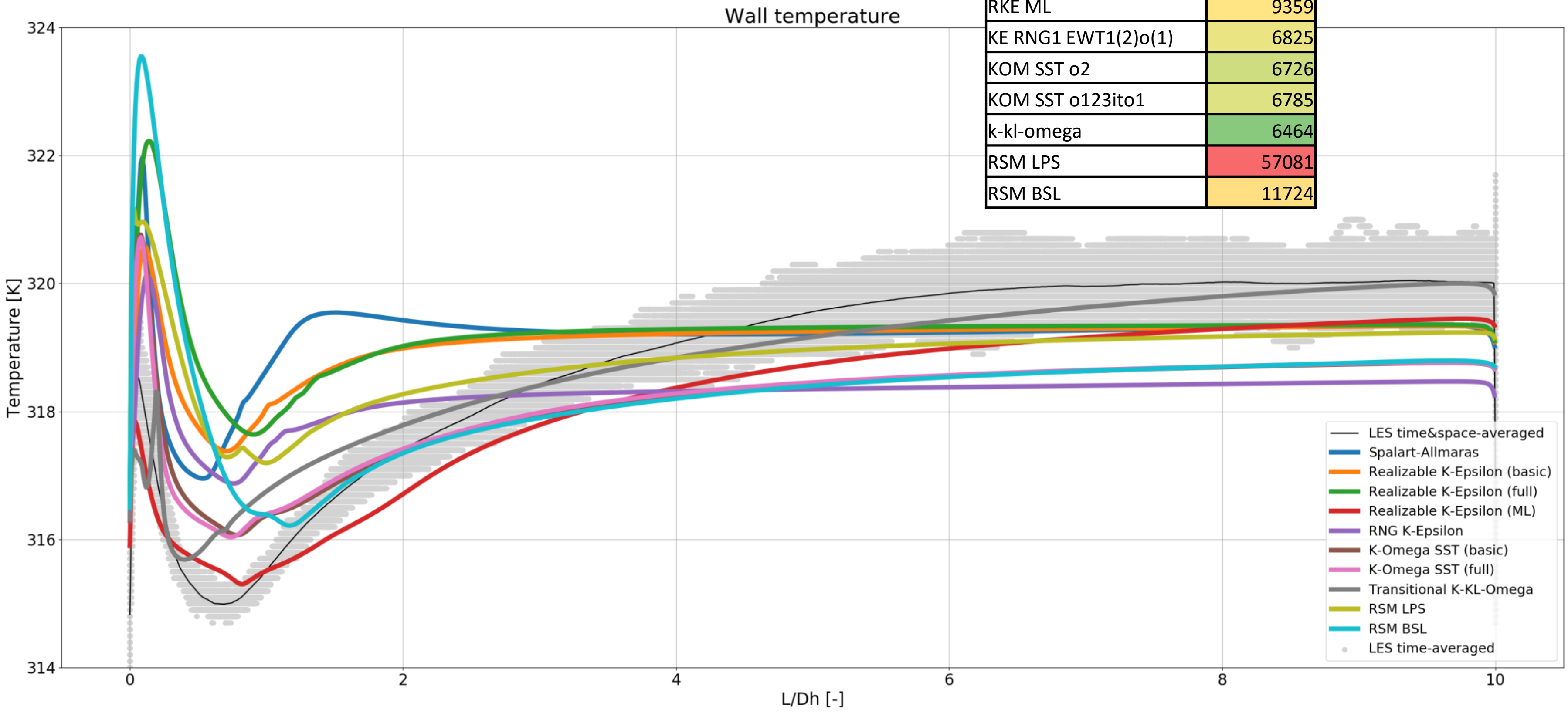
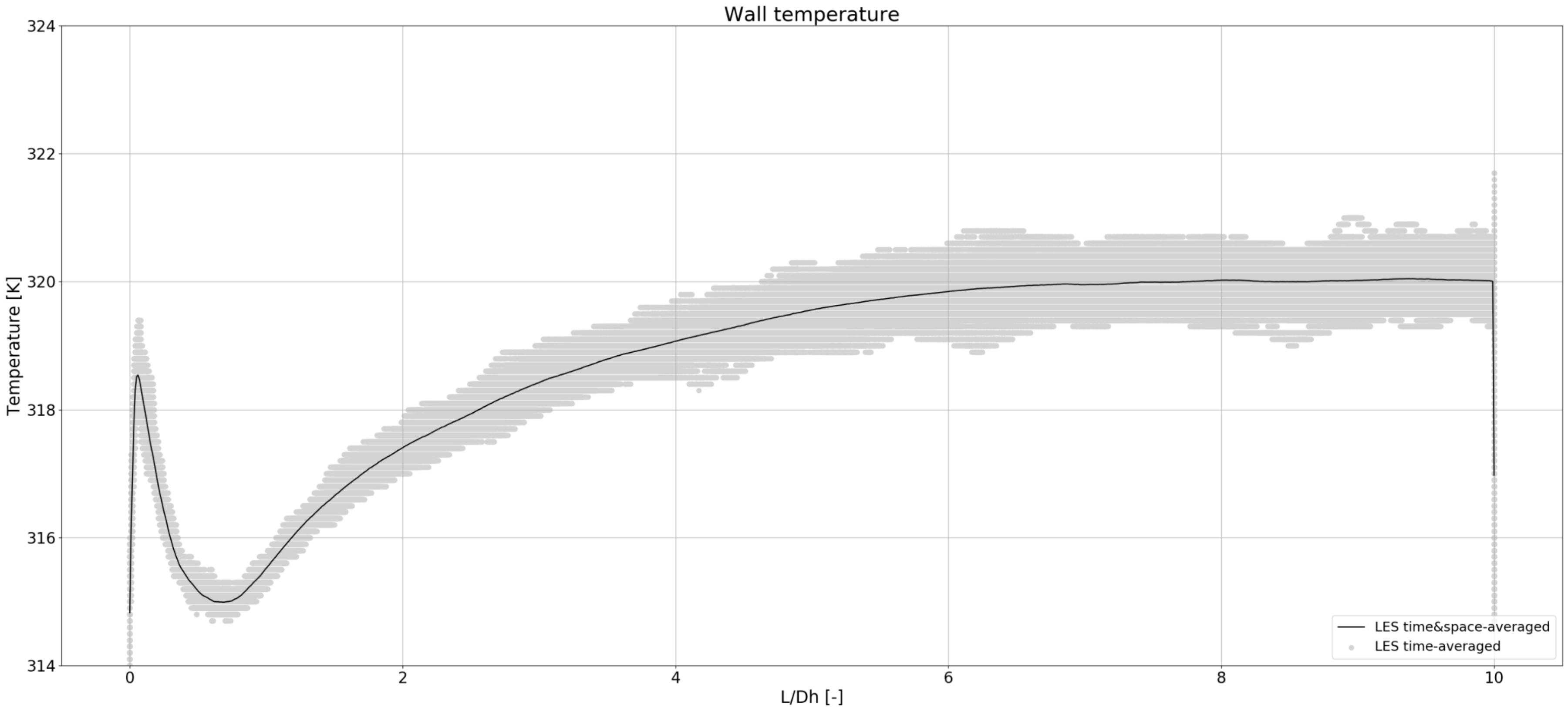


Results spread

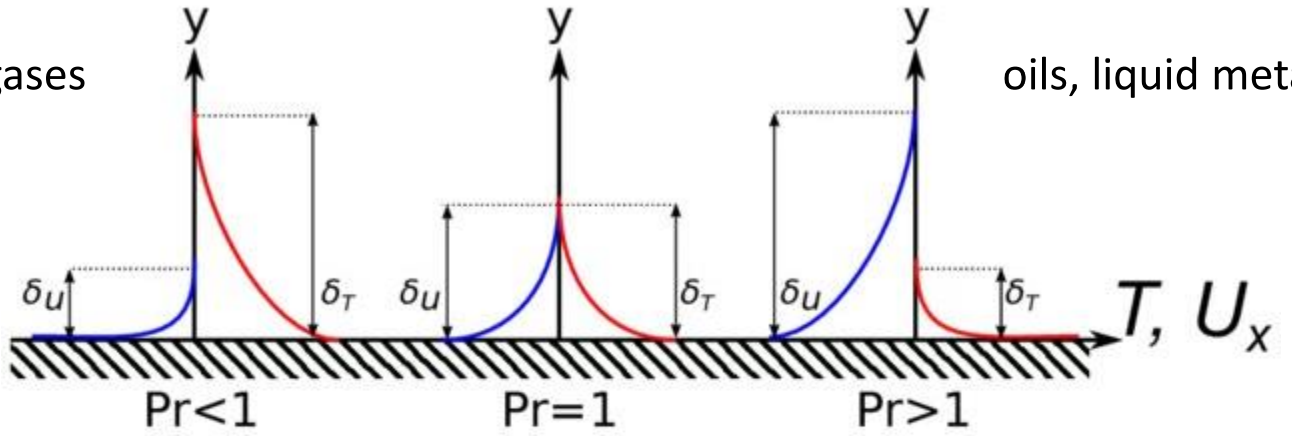


Tricky boundary layer

$$Pr = \frac{c_p \mu}{\lambda}$$

gases

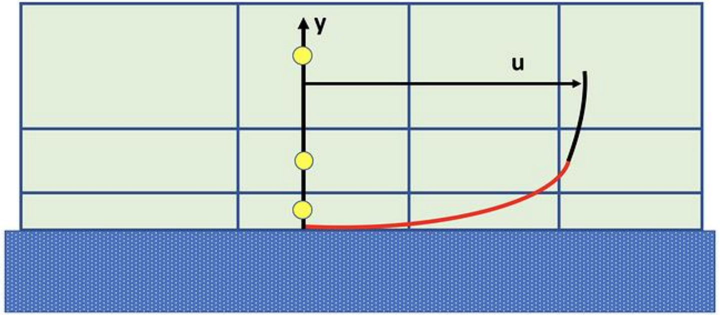
oils, liquid metals



Based on source: Thiele R. (2015), DOI: 10.13140/RG.2.1.4741.5123

Wall functions

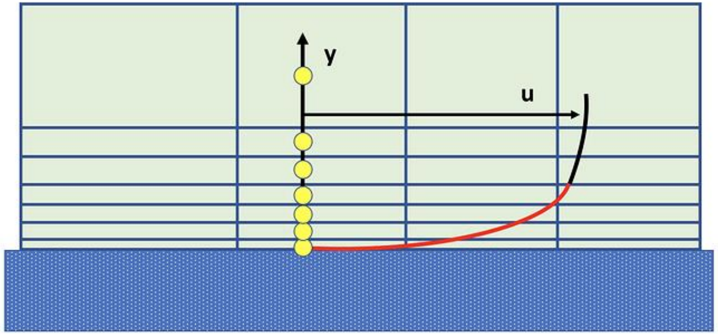
- Logarithmic-based Wall functions to resolve boundary layer



Source: www.simscale.com/forum/t/what-is-y-yplus/82394

- Recommended approach when:
  - High Re flow is to be simulated

- Viscous sublayer resolving approach to resolve boundary layer



- Recommended approach when:
  - Forces on the wall are important
  - Heat transfer
  - Detached flow

Thank you for attention

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